STATS 250 Lab 09

Normal Distribution

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Week of 10/26/2020

Reminders 💡

Your tasks for the week running Friday 10/23 - Friday 10/30:

Task	Due Date	Submission
Lab 9	Friday 10/30 8:00AM ET	Canvas
Homework 6	Friday 10/30 8:00AM ET	course.work

Midterm regrade requests through Gradescope due **Tuesday 10/27 8am ET**

M-Write Prompt 2 opens Wednesday 10/28 at 5pm ET

Weekly Advice

PLEASE PLEASE PLEASE WATCH YOUR PARENTHESES

- Every open parenthesis or bracket must be closed
- Arguments to functions go *inside* parentheses and are separated by commas:
  `abline(v = 1, col = "blue")`

Weekly Advice

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Probability Distributions

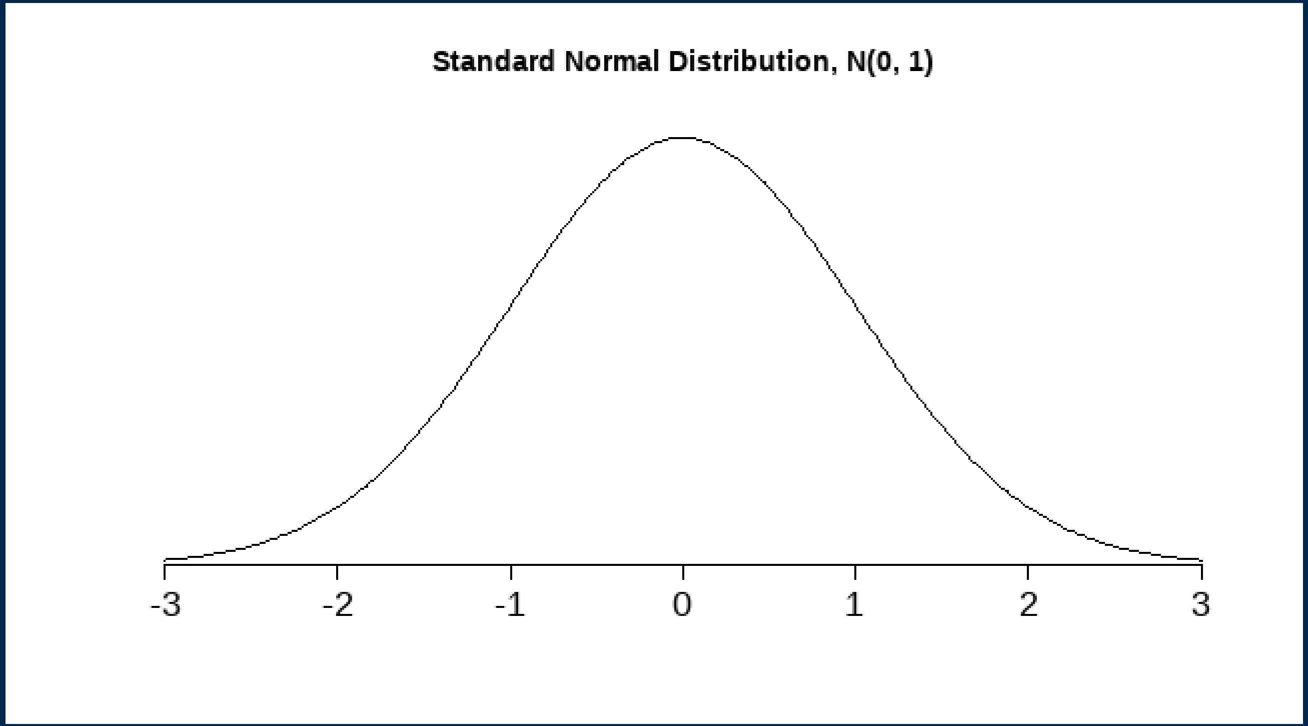
A **distribution** refers to the possible values a random variable can take as well as the probability that it takes those values.

Here's an example distribution of a random variable X:

x_i	0	1	2	3
$P(X = x_i)$	0.5	0.1	0.3	0.1

The Normal Distribution

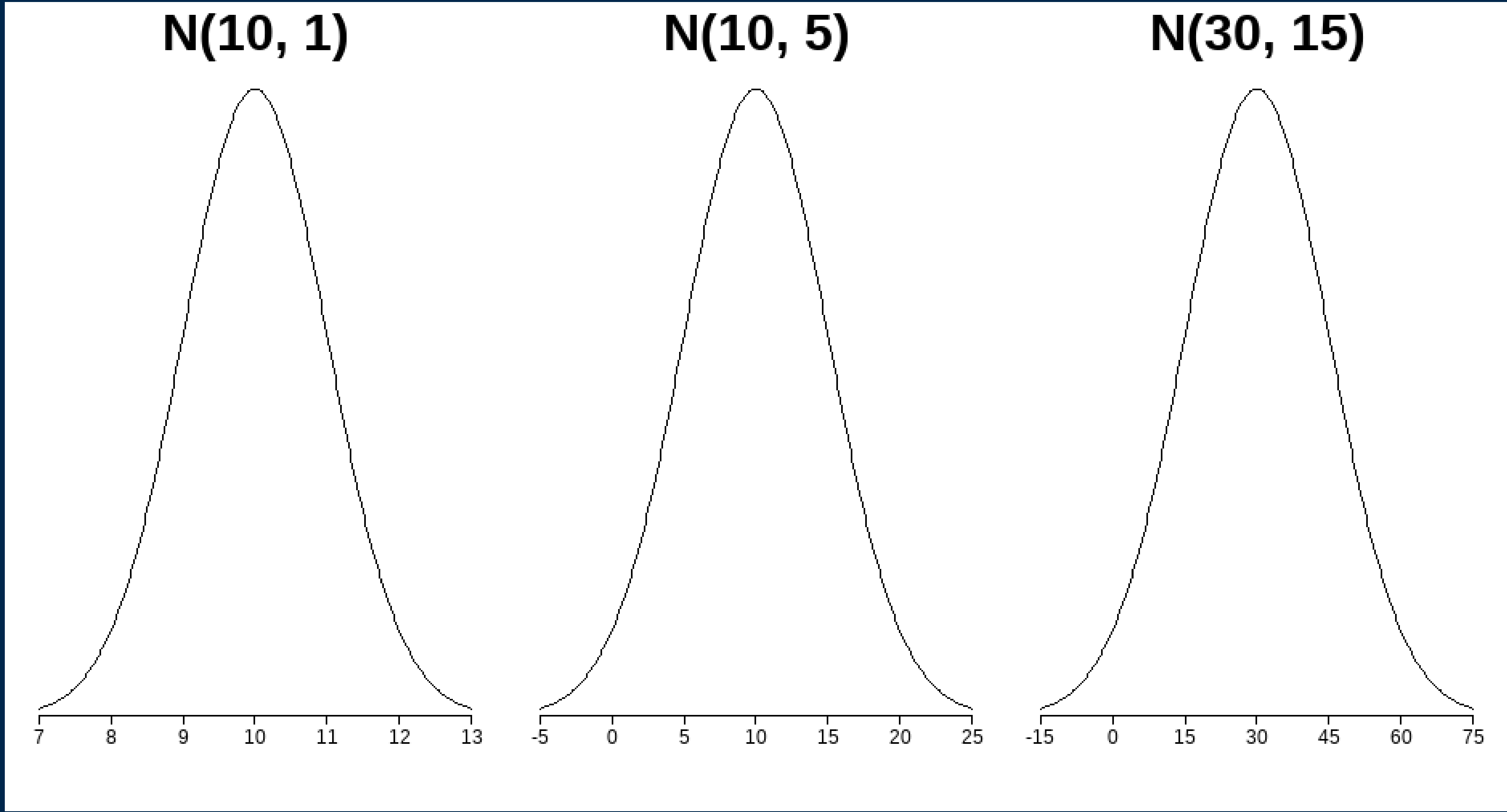
- An *extremely* common distribution in statistics
- Used to describe all sorts of stuff
    - "All models are wrong; some are useful." -George Box



The Normal Distribution

- There are an **infinity** of normal distributions.
- To describe which one we're working with, you need to specify the **mean** and the **standard deviation** of the distribution.
  - Those two numbers *completely describe* the distribution.
- We write $\mathrm{Normal}(\mu, \sigma)$ or $N(\mu, \sigma)$
  - μ (mu) is the population mean
  - σ (sigma) is the population standard deviation

A Selection of Normal Distributions



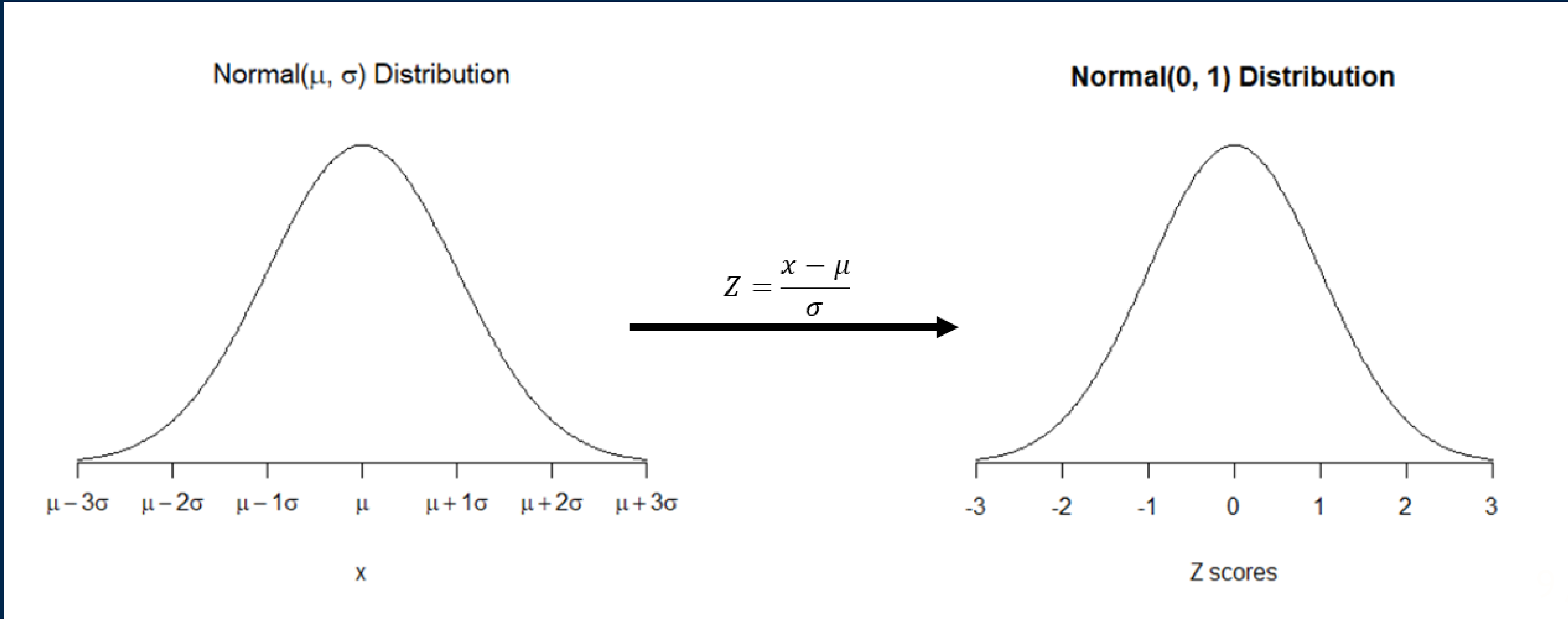
Z Scores

- We can **standardize** a random variable X by subtracting its mean and dividing by the standard deviation.
- If X follows a $N(\mu, \sigma)$ distribution, the standardized version is called a **Z score**.

$$Z = \frac{x - \mu}{\sigma}$$

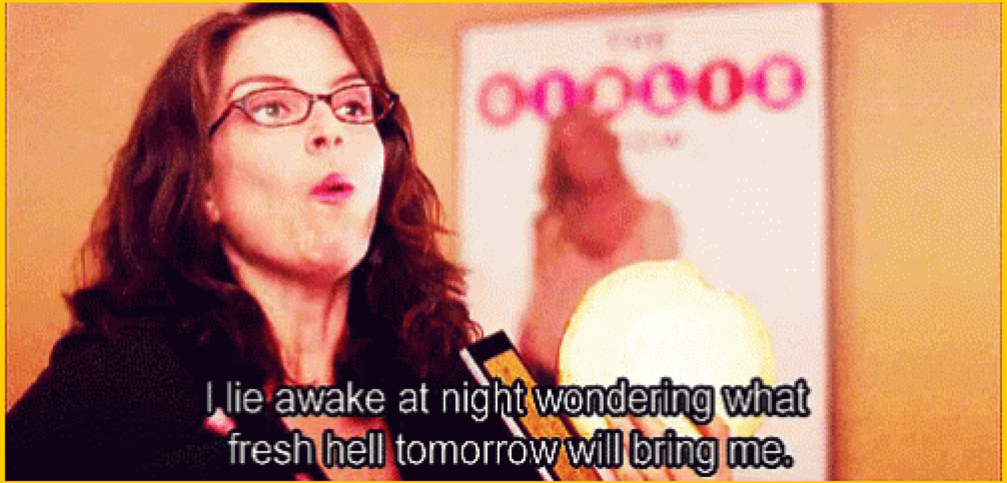
Z Scores

- It's easy to compare things on the same scale, so we standardize.
- Often easier to work with *one* normal distribution: the *standard* normal, $N(0, 1)$.



GIF BREAK!

What questions do you have so far?



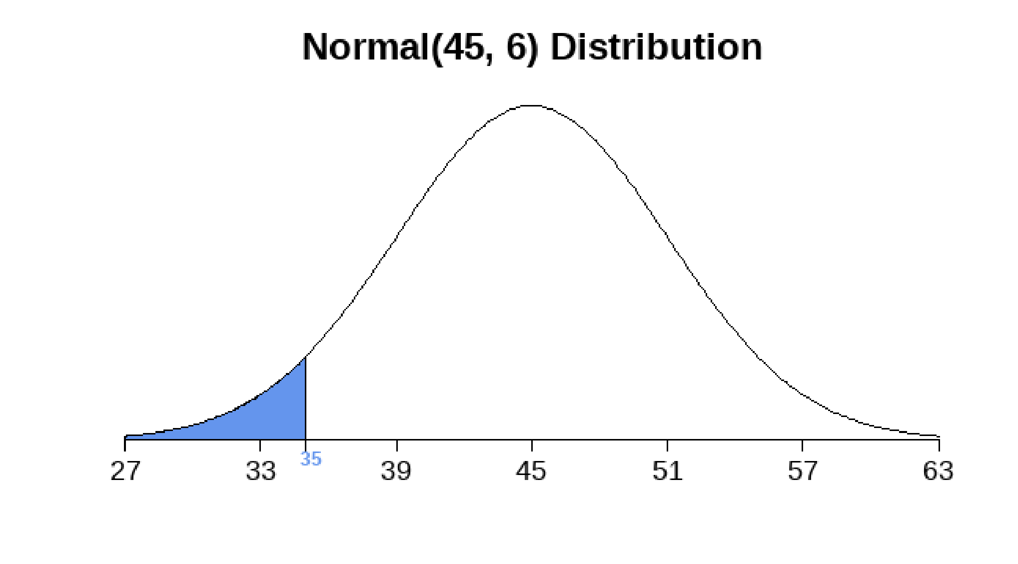
Probability with Normal Distributions

- Given a normally-distributed random variable X, the probability of taking on a certain range of values is the area under the normal curve over those values.
- If X follows a $N(45, 6)$ distribution, $P(X < 35)$ is

```r
plotNorm(mean = 45,
         sd = 6,
         shadeValues = 35,
         direction = "less",
         cex.main = 2)
```

```r
pnorm(q = 35, mean = 45, sd = 6)
```

```
[1] 0.04779035
```



Standardization Works

- Let X have a $N(45, 6)$ distribution. Find $P(X < 35)$.

$$z = \frac{35 - 45}{6} = -1.667$$

```r
pnorm(q = -1.667, mean = 0, sd = 1)
```

```
[1] 0.0477572
```

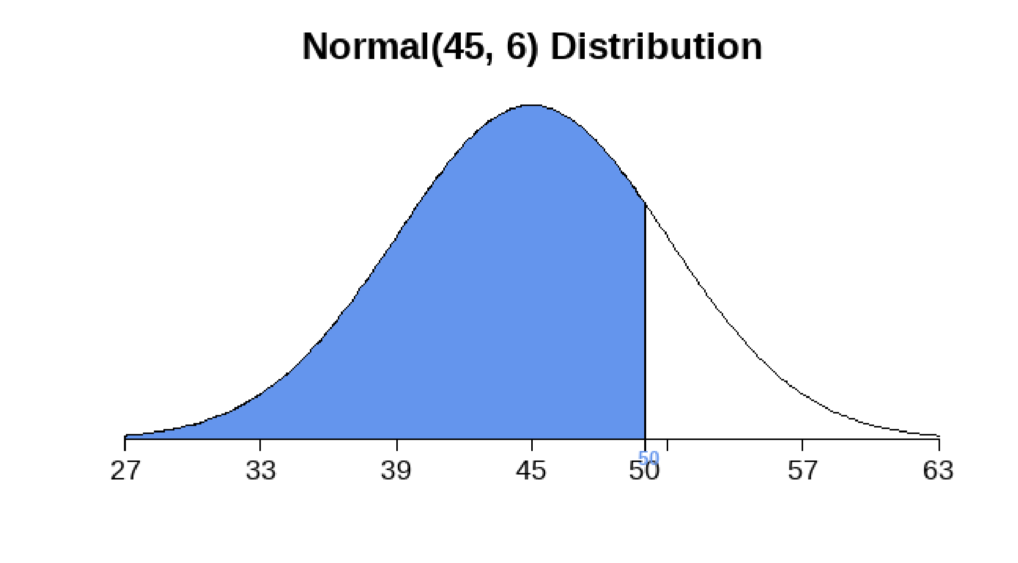
Probability with Normal Distributions

We can also find probabilities of "greater than" events. Again, let X be $N(45, 6)$ and find $P(X > 50)$.

`pnorm()` by default finds *less than* probabilities (area to the *left*)

```
pnorm(50, mean = 45, sd = 6)
```

```
[1] 0.7976716
```



Probability with Normal Distributions

How do we deal with `pnorm()` shading to the left?

```
pnorm(50, mean = 45, sd = 6)
```

```
[1] 0.7976716
```

Strategy 1

Use the fact that the total area under the normal curve is 1:

```
1 - pnorm(50, mean = 45, sd = 6)
```

```
[1] 0.2023284
```

Strategy 2

Set the `lower.tail` argument to `FALSE`:

```
pnorm(50, mean = 45, sd = 6, lower.tail = FALS
```

```
[1] 0.2023284
```

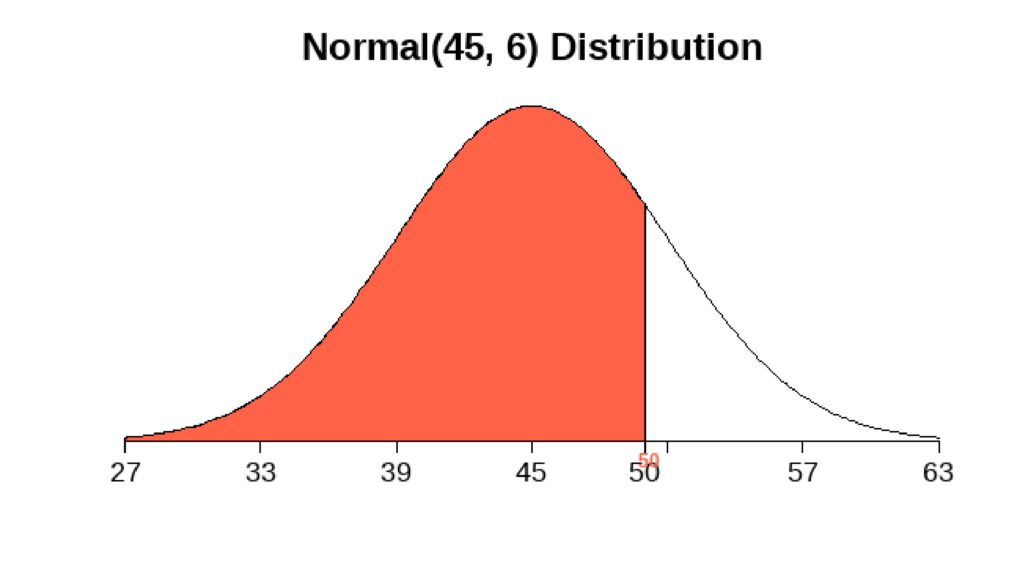
Probability with Normal Distributions

How could we find P(35 < X < 50), again if X has a N(45, 6) distribution?

```
plotNorm(mean = 45, sd = 6,
         shadeValues = 50,
         direction = "less",
         col.shade = "tomato",
         cex.main = 2)
```

```
pnorm(50, mean = 45, sd = 6)
```

```
[1] 0.7976716
```



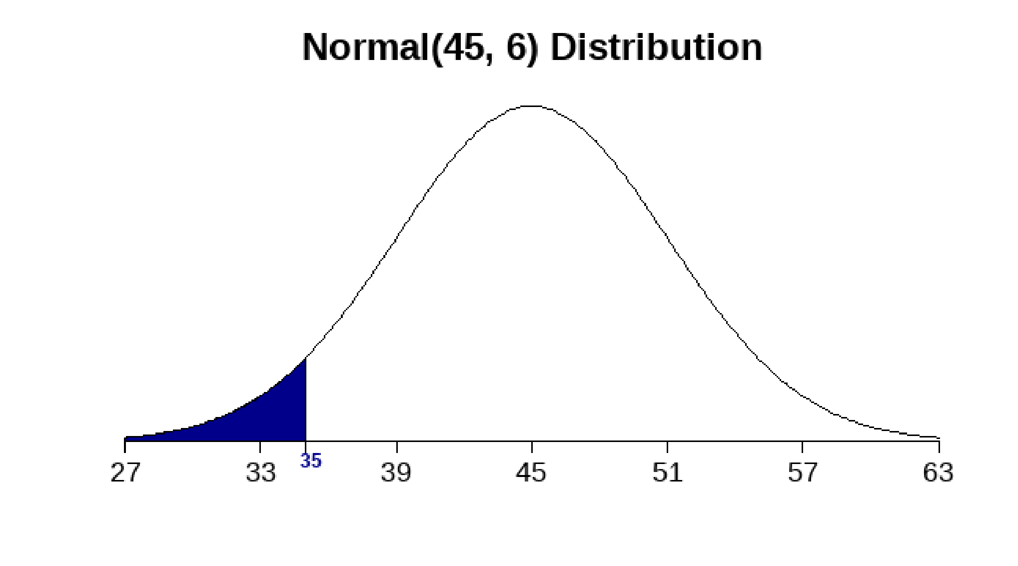
Probability with Normal Distributions

How could we find P(35 < X < 50), again if X has a N(45, 6) distribution?

```r
plotNorm(mean = 45, sd = 6,
         shadeValues = 35,
         direction = "less",
         col.shade = "darkblue",
         cex.main = 2)
```

```r
pnorm(35, mean = 45, sd = 6)
```

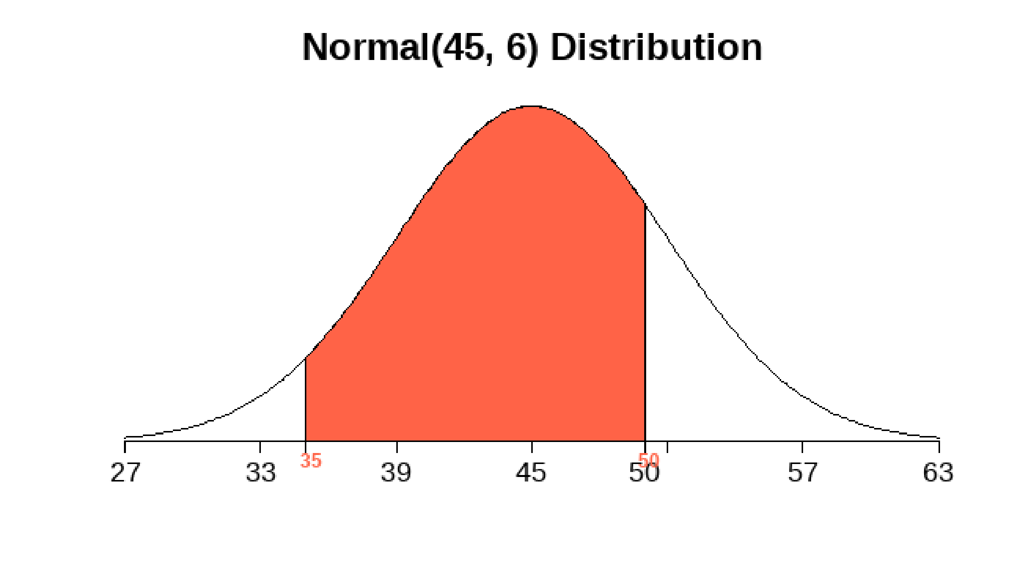
```
[1] 0.04779035
```



Probability with Normal Distributions

How could we find P(35 < X < 50), again if X has a N(45, 6) distribution?

```r
plotNorm(mean = 45, sd = 6,
         shadeValues = c(35, 50),
         direction = "inside",
         col.shade = "tomato",
         cex.main = 2)
```




```r
pnorm(50, mean = 45, sd = 6) -
  pnorm(35, mean = 45, sd = 6)
```

```
[1] 0.7498813
```

GIF BREAK

What questions do you have?

Percentiles of the Normal Distribution

The xth **percentile** of a distribution is the value of a random variable such that x% of the distribution is less than that value.

- Scoring in the 80th percentile on an exam means you got a higher score than 80% of test takers (equivalently, 80% of test takers scored less than you).
- Use `qnorm()` to find percentiles of the normal distribution.

Let's find the 4.8th percentile of the $N(45, 6)$ distribution.

```r
qnorm(p = .048, mean = 45, sd = 6)
```

```
[1] 35.01262
```

Percentiles of the Normal Distribution

Let's find the 30th percentile of the standard normal distribution, N(0, 1).

```
qnorm(p = 0.3) # notice no mean or sd arguments! The defaults are 0 and 1.
```

```
[1] -0.5244005
```

```
qnorm(p = 0.3, mean = 0, sd = 1)
```

```
[1] -0.5244005
```

Type in the chat: Why is this number negative?

Percentiles of the Normal Distribution

`qnorm(p = 0.3, mean = 0, sd = 1)` gives us a Z score! Use this to find the 30th percentile of the $N(45, 6)$ distribution.

$$Z = \frac{x - \mu}{\sigma}$$

Take a minute to fill in the chunks on line 163 and 171.

Percentiles of the Normal Distribution

`qnorm(p = 0.3, mean = 0, sd = 1)` gives us a Z score! Use this to find the 30th percentile of the $N(45, 6)$ distribution.

$$Z = \frac{x - \mu}{\sigma}$$

Take a minute to fill in the chunks on line 163 and 171.

```
qnorm(0.3) * 6 + 45
```

```
[1] 41.8536
```

```
qnorm(0.3, mean = 45, sd = 6)
```

```
[1] 41.8536
```

Code Cheat Sheet 💻

`pnorm(q, mean = 0, sd = 1, lower.tail = TRUE)`

- **`q`** refers to the value you want to find the area above or below
  - `pnorm(q, 0, 1)` gives $P(Z < q)$ where Z is $N(0, 1)$
- **`mean`** refers to μ, defaults to 0
- **`sd`** refers to σ, defaults to 1
- **`lower.tail`** controls which direction to "shade": `lower.tail = TRUE` goes less than q, `lower.tail = FALSE` goes greater than q; defaults to TRUE

Code Cheat Sheet 💻

`qnorm(p, mean = 0, sd = 1, lower.tail = TRUE)`

- **`p`** refers to the area under the curve
    - `qnorm(p, 0, 1)` is the number such that the area to the left of it is p
- **`mean`** refers to μ, defaults to 0
- **`sd`** refers to σ, defaults to 1
- **`lower.tail`** controls which direction to "shade": `lower.tail = TRUE` goes less than q, `lower.tail = FALSE` goes greater than q; defaults to TRUE

Code Cheat Sheet 💻

`plotNorm(mean = 0, sd = 1, shadeValues, direction, col.shade, ...)`

- **`mean`** refers to μ, defaults to 0
- **`sd`** refers to σ, defaults to 1
- **`shadeValues`** is a vector of up to 2 numbers that define the region you want to shade
- **`direction`** can be one of `less`, `greater`, `outside`, or `inside`, and controls the direction of shading between `shadeValues`. Must be `less` or `greater` if `shadeValues` has only one element; `outside` or `inside` if two
- **`col.shade`** controls the color of the shaded region, defaults to `"cornflowerblue"`
- **`...`** lets you specify other graphical parameters to control the appearance of the normal curve (e.g., `lwd`, `lty`, `col`, etc.)

Lab Project ⌨️

Your tasks

- Complete the "Try It!" and "Dive Deeper" portions of the lab assignment by copy/pasting and modifying appropriate code from earlier in the document.
- Introduce yourself to your collaborators
- **Do not leave people behind.**

How to get help

- Ask your collaborators -- share your screen!
- Use the "Ask for Help" button to flag me down.

Reminders

http://bit.ly/250ticket9

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